



Chapter 9: Rotational Dynamics

Essential Concepts and Summary

Action of Forces and Torques on Rigid Objects

- ◆ Magnitude of torque = (Magnitude of force) times (Lever arm)

$$\tau = F l$$

- ◆ SI unit: newton*meter
- ◆ The torque is positive when the force tends to produce a counterclockwise rotation about the axis, and negative when it produces clockwise rotation

Rigid Objects in Equilibrium

- ◆ A rigid body is in equilibrium if it has zero translational acceleration and zero angular acceleration. In equilibrium, the sum of the externally applied forces is zero, and the sum of the externally applied torques is zero.

$$\sum F = 0$$

$$\sum \tau = 0$$

Reasoning Strategy

- ◆ 1. Select object to which equations are to be applied
- ◆ 2. Draw a free-body diagram
- ◆ 3. Choose a convenient set of axes, and resolve all forces into components lying along axes.
- ◆ 4. Apply equations specifying external forces = 0
- ◆ 5. Select a convenient axis of rotation, identify the point where each external force acts on the object, and calculate the torque produced by each force.
- ◆ 6. Set the sum of the torques equal to 0

Center of Gravity

- ◆ The center of gravity of a rigid object is the point at which its weight can be considered to act when the torque due to the weight is being calculated
- ◆ It is a weighted average of the sum of the object's weight and mass distribution

$$x_{cg} = \frac{W_1 x_1 + W_2 x_2 + \dots}{W_1 + W_2 + \dots}$$

Newton's Second Law For Rotational Motion About a Fixed Axis

- ◆ The moment of inertia I of a body composed of N particles, where m is the mass of a particle and r is the perpendicular distance of the particle from the axis of rotation, is:

$$I = m_1 r_1^2 + m_2 r_2^2 + \dots m_N r_N^2 = \sum mr^2$$

- ◆ Newton's second law for rotational motion for rigid body about a fixed axis, where τ is net external torque, I is the moment of inertia, and α is its angular acceleration, is:

$$\sum \tau = I\alpha$$

Rotational Work and Energy

- ◆ Rotational work done by a constant torque in turning a rigid body through an angle is:

$$W_R = \tau\theta$$

- ◆ Rotational kinetic energy of a rigid object rotating with angular speed about a fixed axis is:

$$KE_R = \frac{1}{2} I\omega^2$$

- ◆ Total mechanical energy of a rigid rotating body is the sum of its rotational, translational, and kinetic energies:

$$E = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 + mgh$$

Angular Momentum

- ◆ Angular momentum of a rigid body rotating with an angular momentum about a fixed axis and having a momentum of inertia I (where angular momentum is L) is:
- ◆ Principle of conservation of angular momentum states that the total angular momentum of a system remains constant as long as the net average external torque is zero.

$$L = I\omega$$

$$\Delta(I\omega) = 0$$

$$\Delta(mvr) = 0$$