10

Conservation of Momentum

Billiard Ball Physics

Purpose

To investigate conservation of momentum and energy of two colliding pendula.

Required Equipment and Supplies

Colliding Pendula Apparatus from SciMaTech
spark timer (variable frequency)
thermal spark paper
polar plot paper (optional)

Discussion

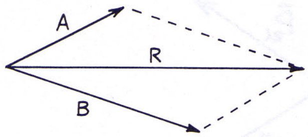
Ever break a rack of billiard balls and wonder how they were going to spread on the table after the balls rolled to a stop? A pool shark may have an entirely different explanation than a physicist, but the *conservation of momentum* principle is fundamental to any explanation. Momentum conservation is a result of Newton's Second Law. Newton's Second Law, $F = ma$, can be rewritten as

$$F = m\frac{\Delta v}{\Delta t}$$

Multiplying both sides by the time interval Δt we get

$$F\Delta t = m\Delta v = m(v_{\text{final}} - v_{\text{initial}}) = \Delta(mv)$$

The product of mass and velocity, mv, is *momentum*. The quantity $F\Delta t$ is *impulse*. Impulse equals the *change in momentum*, Δmv. In any interaction, whether it be billiard balls colliding or atoms bouncing off one another, if the net external force is zero, then the *change* in momentum is zero.



$$F\Delta t = \Delta(mv) = 0$$

$$0 = mv_{\text{final}} - mv_{\text{initial}}$$

So we see that $mv_{\text{final}} = mv_{\text{initial}}$

Thus, if the net external force on a *system* is zero ($F = 0$), the *change in momentum is zero*. Momentum before interaction is equal to the momentum after interaction. Momentum has been transferred between different parts of the interacting system, with none lost and none gained. This is the idea of the conservation of momentum.

In this experiment you will release two pendulum bobs of equal mass, allow them to collide, and measure their momenta before and after the collision. You will use a spark timer to measure the position of each bob after a succession of equal time intervals. At the bottom of their swing where motion is horizontal, the momenta of the bobs is independent of the vertical force of gravity acting on them. So with no external force in the

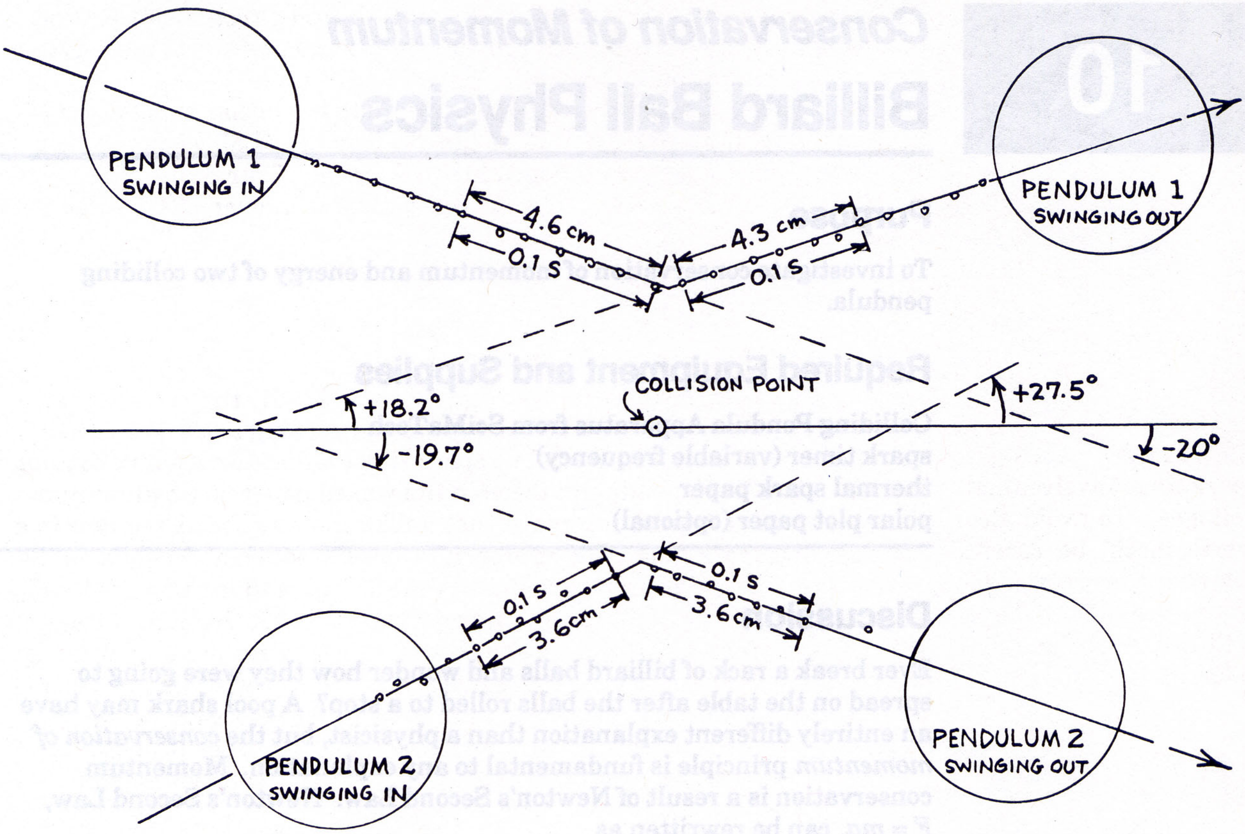


Figure 10.1

direction of their momenta, no momentum is lost or gained when they collide.

Suppose the balls swing towards each other, collide, and rebound at the angles and speed as shown in Figure 10.1. Suppose the initial momenta of pendulum-1 and pendulum-2 are p_{1i} and p_{2i} as shown in Figure 10.2. Sketch the resultant initial momentum vector and label it $p_{initial}$. Likewise, the final momenta of pendulum-1 and pendulum-2 are p_{1f} and p_{2f} as shown in Figure 10.3. Sketch the resultant final momentum vector and label it

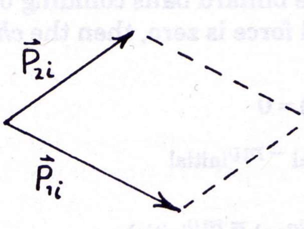


Figure 10.2

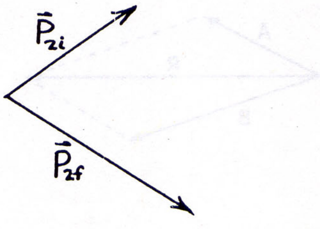


Figure 10.4

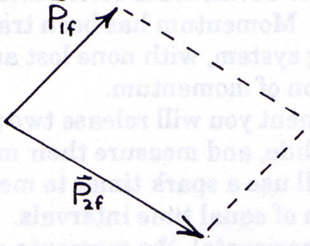


Figure 10.3

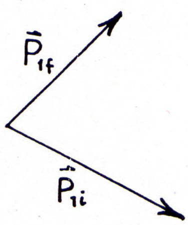


Figure 10.5

p_{final}. What do you notice about the magnitude and direction of the total momentum *before* the collision of the pendula compared to the total momentum *after* the collision?

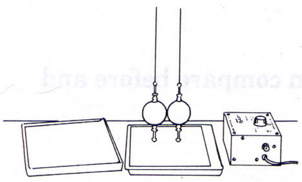
Now you can compare the final momentum to the initial momentum of each ball by subtracting p_{1f} from p_{1i} and p_{2f} from p_{2i}. Label your resultants as the change in momentum of pendulum-1, Δp_1 in Figure 10.4, and Δp_2 for pendulum-2 in Figure 10.5. How do the change in momenta of the pendula compare?

Procedure

Collision of Two Equal Masses

CAUTION: Using a spark timer involves high voltages. To avoid electrical shock, be careful and obey the precautions of the manufacturer as well as those of your instructor.

Step 1. Record the masses of the pendula (essentially the mass of the billiard balls). Set up the pendula apparatus according to the instructions supplied by the manufacturer. The spark timer will cause a spark to jump from the tip of each pendulum through the waxed paper on an aluminum data platform at regular intervals—not unlike the way the distributor in a car does in a spark plug. The appropriate spark frequency depends on the speed of the pendula. Ten hertz usually works well and is computationally convenient, but may yield too few data points. Connect the high voltage and ground wires from the spark timer to the two loose wires hanging from the swivels. Anchor the wires so that there is a minimum of horizontal tension on the suspension, and there is no interference with the swing of the pendula. It may be necessary to adjust the gap between the tip of the pendulum and the data platform for optimum performance. Ask your instructor for assistance if you encounter difficulties.



Step 2. Remove the plastic from the paper hold-downs at the corners of the data platform and lock a thermal sheet onto it. Practice releasing the pendula so that the collision of the pendula causes each pendulum to change its direction at least 25 degrees. Just as the pendula are released, signal your partner to turn on the spark timer. Adjust the frequency of the spark timer so that there are several dots in the 3 or 4 centimeters before and after the collision. Use those 3 or 4 centimeters to determine the speed of the pendula before and after the collision and the entire track to determine their direction. Repeat until you get a good clean set of tracks of the pendula.

Step 3. To measure the velocities, draw the best straight line through each track. Draw a reference axis on your data sheet and measure the direction of the tracks. Compute the speeds by measuring the distance traveled in an integral number of time intervals and divide by the elapsed time. The elapsed time is the rate of the spark timer multiplied by the number of intervals. For example, if the distance between seven dots is 4.6 cm and the frequency is 60 Hz, the speed is 4.6 cm/(6 x $\frac{1}{60}$)s or 46 cm/s. Calculate speeds and measure their directions for all four tracks. Record your data in Data Table 10.1.

Step 4. Using your data from Data Table 10.1, calculate the momentum of each pendulum before and after the collision and record your results in Data Table 10.2.

Step 5. Make a vector diagram of the initial and final momentum of each pendulum on polar graph paper. Find the *sum* and *difference* in momenta of the pendula *before* and *after* the collision. Label the differences in momenta Δp_1 and Δp_2 and the sums $(p_1 + p_2)_{before}$ and $(p_1 + p_2)_{after}$ for each pendulum respectively.

Data Table 10.1

MASS OF PENDULUM-1 m_1 =		MASS OF PENDULUM-2 m_2 =	
BEFORE COLLISION		AFTER COLLISION	
SPEED	DIRECTION	SPEED	DIRECTION

Data Table 10.2

MOMENTUM BEFORE		MOMENTUM AFTER	
$m_{1b}v_{1b}$	$m_{2b}v_{2b}$	$m_{1a}v_{1a}$	$m_{2a}v_{2a}$

Analysis

1. How does the difference or *change in momentum* of each pendulum compare before and after the collision?

2. How does the *sum* of each pendulum's momentum compare before and after the collision?

3. Is the momentum of the system of the two balls conserved?

Going Further

Collision of Two Unequal Masses

Increase the mass of one of the pendula by unscrewing the nuts on top of the pendulum bob and inserting two 100-g masses, and repeat the experiment for two unequal masses. Compare your results to that of equal masses.

If time does not allow doing the experiment quantitatively, try doing it qualitatively. Predict the results and then try it and see.

Inelastic Collisions

Repeat the experiment for two equal masses with a slip of Velcro attached to each pendulum bob. What do you observe?