Projectile Motion

Bull's Eye

Purpose
To investigate the independence of horizontal and vertical components of motion. To predict the landing point of a projectile.

Required Equipment and Supplies
ramp
1/2” (or larger) steel ball
empty soup can
meterstick
plumb line
Apple II Series computer
interface box with photogates
light sources (2 D-cell flashlights work well)
"Chronometer" timing program from LabTools
"Bull's Eye" simulation (optional)

Discussion
If you tossed a rock in some region of gravity-free outer space, it would just keep going—forever—in accordance with Newton’s first law. The rock would continue its motion at constant velocity and cover a constant distance each second (Figure 11.1). When motion is uniform, the equation for distance traveled is

$$\Delta d = v \Delta t$$

and the speed is

$$v = \frac{\Delta d}{\Delta t}$$

Back on earth, what happens when you drop a rock? It falls to the ground and the distance it covers in each second increases (Figure 11.2). Gravitation is constantly increasing the rock’s speed. If we let $\Delta y$ represent vertical distance (and $x$ horizontal distance) then the equation of the vertical distance fallen by an object initially at rest in $t$ seconds is:

$$\Delta y = \frac{1}{2} g (\Delta t)^2$$

where $g$ is the free-fall acceleration. Starting from rest, the speed gained, $\Delta v$, after time $\Delta t$ is

$$\Delta v = g \Delta t$$
What happens when you toss the rock horizontally (Figure 11.3)? The curved motion that results can be described as the combination of two straight-line components of motion: one vertical and the other horizontal. The vertical component undergoes acceleration toward the center of the earth, while the horizontal component does not. The secret to analyzing projectile motion is to keep two separate sets of "books"—one that treats the horizontal motion according to

\[ \Delta x = v \Delta t \]

and the other that treats the vertical motion according to

\[ \Delta y = \frac{1}{2} g (\Delta t)^2 \]

**Horizontal Motion**
- When thinking about how far, think \( x = vt \).
- When thinking about how fast, think \( v = \frac{x}{t} \).

**Vertical Motion**
- When thinking about how far, think \( y = \frac{1}{2}gt^2 \).
- When thinking about how fast, think \( v = gt \).

Your goal in this experiment is to predict where a steel ball will land when released from a certain height on an incline. When engineers build bridges or skyscrapers they do not do so by trial and error. For the sake of safety and economy, it must be right the first time. Computer simulations enable engineers to check and double check their calculations. The "Bull's Eye" simulation will allow you to enter your measurements and test your prediction on the computer before you actually run the experiment. The final test of your measurements and computations will be to position an empty soup can so that the ball lands in the can the first time.

**Procedure**

**Step 1.** Assemble your ramp. Make it as sturdy as possible so the steel balls roll smoothly and reproducibly, as shown in Figure 11.4. The ramp should not sway or bend. The ball must leave the table horizontally. Make the horizontal part of the ramp at least 20 cm long. The vertical height of the ramp should be at least 40 cm.
Step 2. Use a photogate to measure the time it takes the ball to travel from the first moment it reaches the level of the table top (point A in Figure 11.4) to the time it leaves the table top (point B in Figure 11.4). Divide this time interval into the horizontal distance on the ramp (from point A to point B) to find the horizontal speed. Release the ball from the same point (marked with tape) on the ramp several times so that your timings are consistent. Do not let the ball strike the floor! Record the average horizontal speed.

horizontal speed, $v_x =$ cm/s

Step 3. Using a plumb line and a string, measure the vertical distance $y$ the ball must drop from the bottom end of the ramp in order to land in an empty soup can on the floor.

1. Should the height of the can be taken into account when measuring the vertical distance $y$? If so, make your measurements accordingly.

Step 4. Using the appropriate equation from the discussion, find the time $t$ it takes the ball to fall from the bottom end of the ramp and land in the can. Write the equation that relates $y$ and $t$ and solve for $t$.

equation for the vertical distance:

$t =$

Step 5. The range is the horizontal distance a projectile travels, $x$. Predict the range of the ball. Write the equation you used to predict the range. Write down your predicted range.

equation for the range:

predicted range: $x =$

Place the can on the floor where you predict it will catch the ball.

Step 6. After your instructor has checked your predicted range and your can placement, release the ball from the marked point on the ramp.

Analysis

2. Did the ball land in the can?

3. What may cause the ball to miss the target? Would your reason increase or decrease the range?
4. You probably noticed that the range of the ball increased in direct proportion to the speed at which it left the ramp. The speed depends on the release point of the ball on the ramp. What role do you think air resistance had in this experiment?

5. What was your percentage error? In what direction is the error most likely to be?