Derivation of the Exchange of Velocities

For a Perfectly Elastic Collision Between Balls of the Same Mass

Variables

m : The mass of a ball

v_{1i}: Velocity of ball 1 before collision

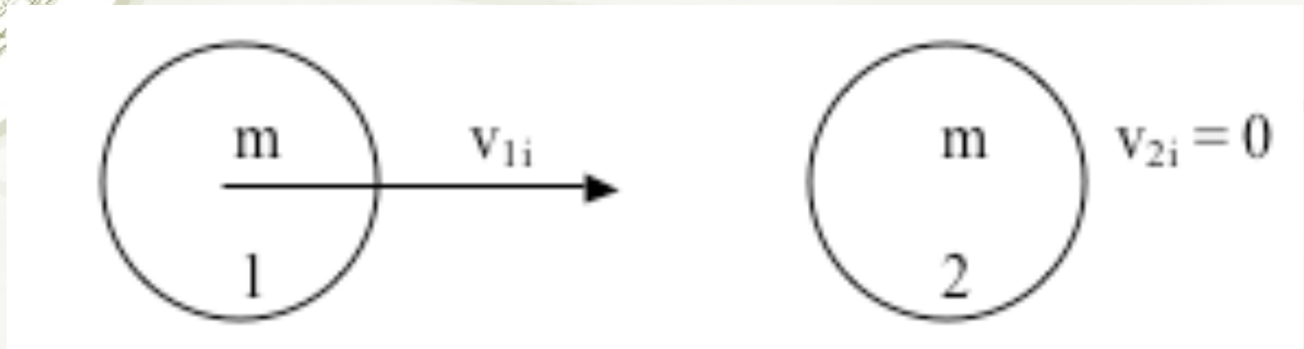
v_{2i}: Velocity of ball 2 before collision

v_{1f}: Velocity of ball 1 after collision

v_{2f}: Velocity of ball 2 after collision

Consider the elastic collision of two balls of equal mass

Before Collision



Momentum Before: mv_{1i}

Kinetic Energy Before: $\frac{1}{2}mv_{1i}^2$

Consider the motion of the balls after the collision

After Collision

Momentum After: $mv_{1f} + mv_{2f}$

Kinetic Energy After: $\frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2$

In any collision, momentum is conserved

$$mv_{1i} = mv_{1f} + mv_{2f}$$

In an elastic collision, Kinetic Energy is conserved

$$\frac{1}{2}mv_{1i}^2 = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2$$

Solve for v_{1f} *and* v_{2f}

$$mv_{1i} = mv_{1f} + mv_{2f}$$

(mv conservation)

$$\frac{1}{2}mv_{1i}^2 = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2$$

(energy conservation)

$$v_{1i} = v_{1f} + v_{2f}$$

$$v_{1i}^2 = v_{1f}^2 + v_{2f}^2$$

$$v_{1i} = v_{1f} + v_{2f}$$

$$v_{1i}^2 = (v_{1f} + v_{2f})^2$$

$$= v_{1f}^2 + 2v_{1f}v_{2f} + v_{2f}^2$$

but since . . .

$$v_{1i}^2 = v_{1f}^2 + v_{2f}^2 = v_{1f}^2 + 2v_{1f}v_{2f} + v_{2f}^2$$

means

$$2v_{1f}v_{2f} = 0$$

so that either

$$v_{1f} = 0 \qquad \text{or} \qquad v_{2f} = 0$$

Since the second ball cannot have zero velocity after the collision, we see that

$$v_{2f} \neq 0$$

Therefore $v_{1f} = 0$

$$v_{1i} = v_{1f} + v_{2f}$$

$$v_{1i} = 0 + v_{2f}$$

$$v_{1i} = v_{2f}$$

Since the balls have the same mass, the velocity (and energy) is transferred from the first ball to the second—that is, they “exchange” velocities. When the balls have different masses, the situation becomes more complicated!