8 Non-Free Fall

Impact Speed

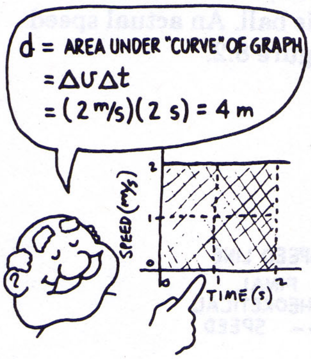
Purpose

To estimate the speed of a falling object as it strikes the ground.

Required Equipment and Supplies

string with rock tied to one end
object with a large drag coefficient such as a leaf, feather, Styrofoam ball, Whiffle ball, tennis ball, golf ball, marble, or baseball
Apple II Series computer (optional)
"Impact Speed" simulation (optional)

Discussion



The area of a rectangle is found by simply multiplying its height by its base. The area of a triangle is equal to its height multiplied by half its base. If the height and the base are measured in meters, the area is measured in square meters. Consider the area under a graph of speed vs. time. The height represents the speed measured in meters per second, m/s, and the base represents time measured in seconds, s. The area of the patch is the speed times the time, expressed in units of m/s · s, which equals meters. Speed multiplied by time is the distance traveled. So the area under a speed-vs-time graph represents the distance traveled. This very powerful idea underlies the mathematics of integral calculus. You will investigate this idea and see how it can be used to predict the distance covered by an object falling in the presence of air resistance.

If there is no air friction, a tennis ball or Styrofoam ball will fall at the same constant acceleration, g, so that their change in speed is

$$v_f - v_i = gt$$

where v_f = final speed
v_i = initial speed
g = free fall acceleration
t = time of fall

A graph of speed vs. time of fall is shown in Figure 8.1, where $v_i = 0$. The y-axis represents the speed, v, of the freely falling object at the end of any time, t. The shape bounded by the axes and graphed line is a triangle of base t and height v_f, so the area equals $\frac{1}{2}v_f t$.

To check that this area does represent and equal the distance traveled, note the following. The average speed, v, is half of v_f. The distance, d, traveled by a body undergoing constant acceleration is its average speed, $\bar{v}$, multiplied by the duration, t, of travel.

$$d = \bar{v}t \text{ or } d = \frac{1}{2}v_f t$$

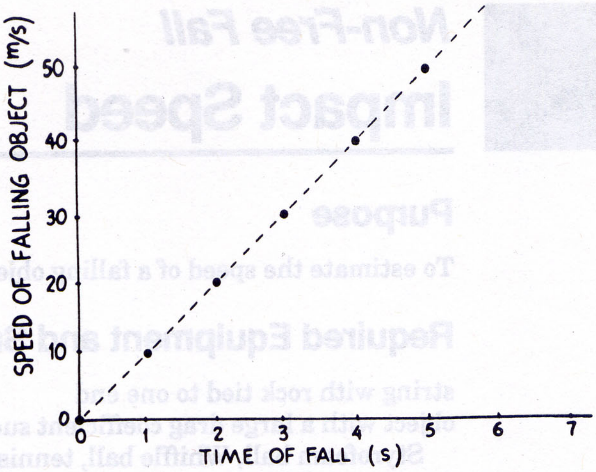


Figure 8.1

In a vacuum, a tennis ball released from rest will fall 43 meters in 2.96 seconds. This is free fall. But falling the same vertical distance in the presence of air will take 3.5 seconds. This is non-free fall. Free fall is approximated for many dense things falling short distances because air friction is negligible compared to the gravitational force. A tennis ball falling many meters, however, is a case of non-free fall, because air friction is not negligible compared to the weight of the tennis ball. An actual speed vs. time graph of a falling tennis ball is shown in Figure 8.2.

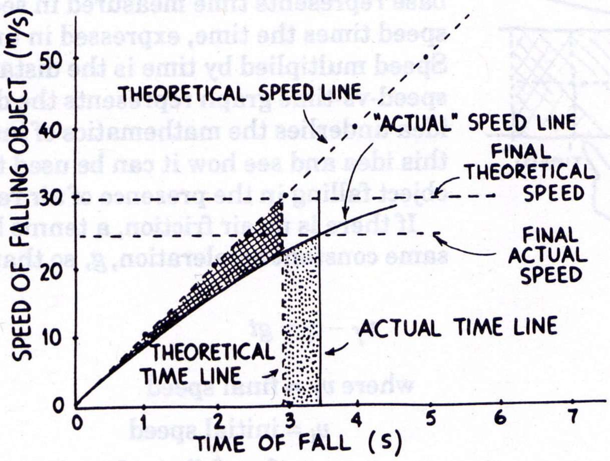
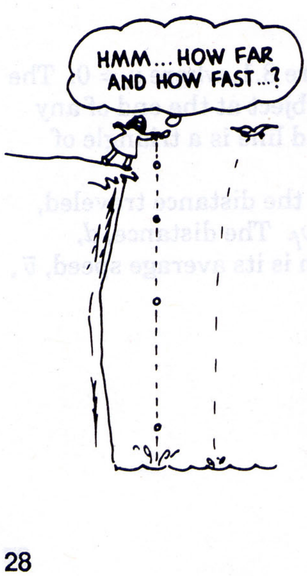


Figure 8.2



The straight line on the graph is the theoretical straight line of free fall. Its constant slope represents the free-fall acceleration, *g*. The curved line shows the actual case, a gradually decreasing slope, which means a gradually decreasing acceleration due to the buildup of air friction as speed increases. Falling speed is less than that of free fall. The difference is small at first, but grows as air resistance builds up. We see that the graph of actual speed vs. time curves beneath the theoretical straight line.

Is there a way to sketch the actual speed vs. time curve from only the distance fallen and the time of fall? The answer is yes. First, for a given height, calculate the theoretical time of impact for the ideal case of falling with no air resistance. Make a plot of speed vs. time for this ideal case. Bound the area under this curve by drawing a vertical line from the calculated time of impact on the horizontal axis up to the straight-line slope. (In Figure 8.2 this line is labeled "theoretical time line.") The bounded area represents the height of drop, which you already know.

If you actually drop a tennis ball from the same height, it will take a *longer* time to make impact because of air resistance. A graph of speed vs. time will have a different shape, for the plotted line will curve beneath the straight line of the free-fall graph. But however much the speed vs. time graph differs from that of the free-fall case, both graphs will have one thing in common—their *areas*. Why? Because the heights are the same for both cases.

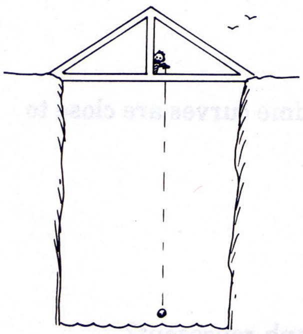
You can sketch a speed vs. time plot for the actual case on the same graph, and similarly draw another vertical line upward from the actual time of impact on the horizontal axis (labeled "actual time line" in Figure 8.2). The main idea is to sketch the curve so you'll bound the same amount of area as the area of the ideal case. In doing this your curve will meet the second vertical line at a point well below the theoretical speed vs. time line. Sketch this curve so that the area added below it due to increased time of fall (stippled area) equals the area subtracted from below the theoretical speed vs. time line due to decreased speed (cross-hatched area). The areas under the two graphs are then equal. This is a fair approximation to the actual curve of speed vs. time. The point where this curve crosses the vertical line of the actual time gives the probable impact speed of the tennis ball.

Procedure

Step 1. Your group should choose a strategy to drop an object and clock its time of fall to within 0.1 s or better. Consider a long-fall drop site, various releasing techniques, and reaction times associated with the timer you use.

Step 2. Devise a method that eliminates as much error as possible to measure the distance the object falls.

Step 3. Submit your plan to your instructor for approval.



Step 4. Measure the height and the falling times for your object using the approved methods of Steps 1 and 2.

height = ________________

actual time of fall = ________________

Step 5. Using your measured value for the height, calculate the theoretical time of fall for your object. Remember, this is the time it would take the object to reach the ground if there were no air resistance.

theoretical time = ________________

Step 6. Using an overhead transparency or graph paper, trace Figure 8.2, leaving out the actual speed vs. time curve. Draw one vertical line from the theoretical time of fall for your height up to the theoretical speed vs. time line. Draw the other vertical line from the actual time of fall up to the theoretical speed vs. time line.

Step 7. Starting from the origin, sketch your approximation for the actual speed vs. time curve, out to the point where it crosses the actual time line, using the example mentioned in the discussion. The area of your stippled region should be the same as that of the cross-hatched region.

One possible way to do this is to tape a piece of cardboard to the wall and project your transparency onto it. Trace your two regions onto the cardboard and cut them out. Then measure the mass of the two regions. If their masses are not the same, adjust your actual speed curve and try again. Your approximation is done when the two regions have the same mass.

Step 8. Draw a horizontal line from the upper right corner of your stippled region over to the speed axis. Where it intersects the speed axis is the object's probable impact speed.

Going Further

Step 9. Follow the directions supplied on the screens from the simulation "Impact Speed." The program utilizes the computational power of the computer to draw speed vs. time graphs so that the areas under the theoretical and actual speed vs. time graphs are equal. The program will allow you to select from two models of friction: 1) drag is proportional to the speed, v, and 2) drag is proportional to the speed squared, v^2.

Enter the mass, height, and falling time of your object. Observe the computer draw the velocity vs. time graphs; first without, and then with drag. Record the computer prediction for the impact speed.

computer prediction = ________

Compare the value for the impact speed as predicted by the computer simulation to your estimation in Step 8. How do they compare?

Analysis

1. Have your instructor overlap your graph with those of others. How does your actual speed vs. time curve compare with theirs?

2. What can you say about objects whose speed vs. time curves are close to the theoretical speed vs. time line?

3. What does the area under your speed vs. time graph represent?

4. The equation for distance traveled is $d = \bar{v}t$. In this lab, the *distance fallen* is the same *with* or *without* air friction. How do the average speeds and times compare with and without air friction? Try to use different-sized symbols such as $mA = Ma$.

5. If you dropped a large leaf from the Empire State Building, what would its speed vs. time graph look like? How might it differ from that of a baseball?

6. The terminal speed of a falling object is the speed at which it stops accelerating. How could you tell whether an object had reached its terminal speed by glancing at an actual speed vs. time graph?