42

Charging of a Capacitor
Getting All Charged Up

Purpose

To determine the time constant for a simple RC circuit.

Required Equipment and Supplies

2 capacitors (25,000-µF and 5,000-µF non-polar Mylar or other high quality capacitors)
2 resistors (1 k-ohm and 2 k-ohm)
6 volt battery
Apple II Series or IBM-compatible computer
Voltage Plotter from Vernier Software *or*
  infinite resistance voltmeter or vacuum tube voltmeter (VTVM)
  stopwatch
graph paper *or*
  Data Plotter graphing program

Note: If the capacitors specified are not readily available, capacitors of smaller capacitance can be used. However, for best results, resistors (or combinations of resistors in series) of correspondingly larger resistance should be employed.

Discussion

When a capacitor is connected to a direct current voltage source such as a battery, current flows and charge builds up on the capacitor plates. The potential difference or voltage across the plates increases until it equals the voltage, V, of the battery. At any time the charge, Q, on one plate of the capacitor is related to the voltage across the capacitor plates by $Q = CV$, where C is the capacitance of the capacitor in farads (F). The rate voltage increases depends on the capacitance of the capacitor and the resistance in the circuit. Similarly, when a charged capacitor is discharged, the rate of voltage "decay" depends on the same parameters.

Both the charging and discharge times of a capacitor are characterized by a quantity called the *time constant*, τ, which is the product of the capacitance, C, and the resistance, R; or $\tau = RC$. In this lab, you will investigate how time constants are related to the charging and discharging characteristics of capacitors.

When a capacitor is charged through a resistor by a direct-current voltage source, the charge on the capacitor and the voltage across the capacitor increase with time. The voltage, V, as a function of time is given by

$$V = V_0\left(1 - e^{-t/RC}\right)$$

where the exponential $e = 2.718...$ is the base of natural logarithms and V_0 is the voltage of the source. The curve of the exponential rise of the voltage with time during the charging process is shown in Figure 42.1.

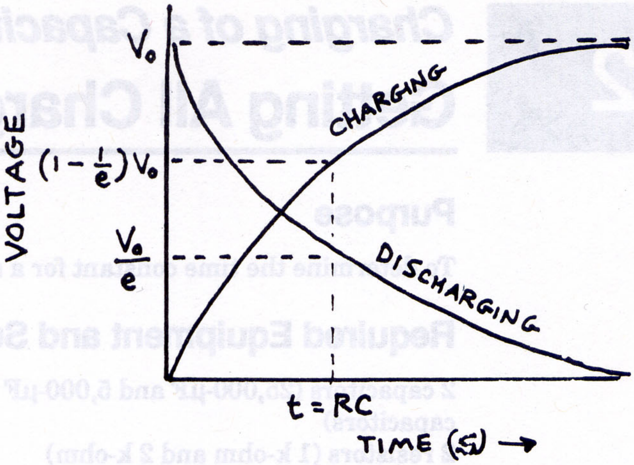

Figure 42.1

After a time $t = \tau = RC$ (one time constant), the voltage across the capacitor has increased to a value of $(1 - 1/e)$ of V_0. That is,

$$
\begin{aligned}
V &= V_0\left(1 - e^{-RC/RC}\right) \\
&= V_0\left(1 - e^{-1}\right) \\
&= V_0\left(1 - \frac{1}{e}\right) \\
&= 0.63V_0
\end{aligned}
$$

When the capacitor is discharged through a resistor, the voltage across the capacitor and the charge on the capacitor decreases or "decays" with time according to the equation

$$V = V_0 e^{-t/RC}$$

The exponential decay of the voltage with time is also illustrated in Figure 42.1. After a time $t = \tau = RC$ (one time constant), the voltage across the capacitor has decreased to a value of $1/e$ of V_0. That is,

$$
\begin{aligned}
V &= V_0 e^{-t/RC} \\
&= V_0 e^{-RC/RC} \\
&= V_0 e^{-1} \\
&= \frac{V_0}{e} \\
&= 0.37V_0
\end{aligned}
$$

We can rearrange the equation for a charging capacitor, $V = V_0\left(1 - e^{-t/RC}\right)$, to get

$$(V_0 - V) = V_0 e^{-t/RC}$$

and taking the natural logarithms of both sides we get

$$\ln(V_0 - V) = \frac{-t}{RC} + \ln V_0$$ Eq. (A)—charging capacitor

Similarly, we can rearrange the equation for a discharging capacitor, $V = V_0 e^{-t/RC}$, to get

$$\ln V = \ln V_0 - \frac{t}{RC}$$

or

$$\ln\left(\frac{V_0}{V}\right) = \frac{t}{RC}$$ Eq. (B)—discharging capacitor

Both Equations A and B can be expressed in the form of the equation for a straight line, $y = mx + b$:

$$\ln(V_0 - V) = \left(\frac{-1}{RC}\right)t + \ln V_0 = \left(-\frac{1}{\tau}\right)t + \ln V_0$$ Eq. (A)—charging capacitor

$$\ln\left(\frac{V_0}{V}\right) = \left(\frac{1}{RC}\right)t = \left(\frac{1}{\tau}\right)t$$ Eq. (B)—discharging capacitor

For a charging capacitor, the slope of the graph of $\ln(V_0 - V)$ vs. t is $-1/RC$, or $-1/\tau$. For a discharging capacitor, the slope of $\ln(V_0/V)$ vs. t is $1/RC$ or $1/\tau$. Hence, the time constants can be experimentally determined by calculating the slopes of $\ln(V_0 - V)$ vs. t and $\ln(V_0/V)$ vs. t graphs.

Procedure

Step 1. Set up the circuit as shown in Figure 42.2 with C_1 (a 25,000-μF capacitor) and R_1 (a 1k-ohm resistor). If a single-pole double-throw switch is not available, leave the ground lead to the battery disconnected. Determine the resistance of the resistor using the color bands on the resistor. (Refer to the color band chart in Figure 42.3. See your instructor if you have questions about interpreting this code.) Record the capacitance of the capacitor and the resistance of the resistor in Data Table 42.1.

Resistance Codes	
Color	**Digit**
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Violet	7
Gray	8
White	9

Figure 42.3

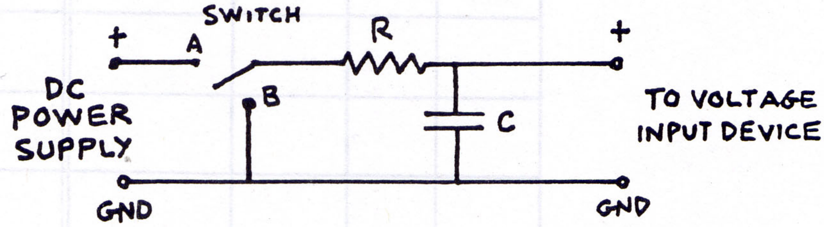

Figure 42.2

Step 2. Connect the voltmeter or the voltage input unit to the computer and the circuit as shown in Figure 42.2. If you are using the computer as a voltmeter, refer to the Teacher's Guide for *Voltage Plotter* or the general instructions for using a Voltage Input Unit or Multi-Purpose Lab Interface. Since the Voltage Input Unit can read a maximum of 3.4 volts, make sure your power supply or battery does not exceed this value. The obvious advantage of the computer is its ability to simultaneously acquire and display the data. Familiarize yourself with the voltmeter or the operation of the computer.

Data Table 42.1

R_1 = ________				C_1 = ________			
CHARGING				DISCHARGING			
V	t	$(V_0 - V)$	$\ln(V_0 - V)$	V	t	V_0/V	$\ln(V_0/V)$

Step 3. Close the switch (or simply connect the ground lead to the battery) to position A and observe the voltage rise of the capacitor on the voltmeter. When the capacitor is fully charged, move the switch to position B and note the voltage decrease as the capacitor discharges. Practice this procedure several times and choose an appropriate sample time based on the charging/discharging time for the capacitor.

Step 4. Have your lab partner simultaneously close the switch to position A as you start the timer. Read and record the capacitor voltage at small time intervals (such as 5–10 seconds) until the capacitor is fully charged (V_0). Record representative data in Data Table 42.2. The computer's ability to take data more frequently with greater precision is very apparent here!

Step 5. After the capacitor is fully charged, open the switch to the neutral position and reset the timer. Then, as before, simultaneously close the switch to position B and start the timer. Read and record the decreasing voltage at small regular time intervals. Open the switch when the capacitor is discharged.

Step 6. Replace R_1 and C_1 with R_2 and C_2 in the circuit (smaller resistance and larger capacitance) and repeat Steps 3, 4, and 5. Record your results in Data Table 42.2.

Analysis

1. Express the units of resistance, ohms, and the units of capacitance, farads, in fundamental units involving charge and time in the space provided below. Use dimensional analysis to show that the product of resistance and capacitance is equivalent to seconds.

$\tau = RC$

(second) = (ohm) (farad)

(second) = ____________

2. Calculate the quantities (V_0–V) and (V_0/V) for the charging and discharging data in both data tables. Using a calculator, find the values of ln(V_0–V) and ln(V_0/V).

3. Make a graph, using *Data Plotter* if available, of ln(V_0–V) vs. time for both sets of data and likewise for ln(V_0/V) vs. time. Draw the straight lines that best fit the data and determine the slope of each line. Record the slopes in the respective data tables. Compute the time constants from the average value of the slope of each graph.

Data Table 42.2

$R_2 =$ ________				$C_2 =$ ________			
CHARGING				DISCHARGING			
V	t	$(V_0 - V)$	$\ln(V_0 - V)$	V	t	V_0/V	$\ln(V_0/V)$

4. Calculate the time constants τ_1 and τ_2 from the known values of the resistance and capacitance.

$\tau_1 =$ ________

$\tau_2 =$ ________

5. How do the slopes of the graphs compare with the values calculated in Question 4? Calculate the percentage difference.