Appendix

Significant Figures and Uncertainty in Measurement

Units of Measurement

All measurements consist of a unit that tells what was measured and a number that tells how many units were measured. Both are necessary. If you say that a friend is going to give you 10, you are telling only *how many*. You also need to tell *what*: ten fingers, ten cents, ten dollars, or ten corny jokes. If your teacher asks you to measure the length of a piece of wood, saying that the answer is 36 is not correct. She or he needs to know whether the length is 36 centimeters, feet, or meters. All measurements must be expressed using a number and an appropriate unit. Units of measurement are more fully covered in your text.

Numbers

Two kinds of numbers are used in science—those that are counted or defined, and those that are measured. There is a great difference between a counted or defined number and a measured number. The exact value of a counted or defined number can be stated, but the exact value of a measured number cannot be known.

For example, you can count the number of chairs in your classroom, the number of fingers on your hand, or the number of nickels in your pocket with absolute certainty. Counted numbers are not subject to error.

Defined numbers are about exact relations, defined to be true. The exact number of centimeters in a meter, the exact number of seconds in an hour, and the exact number of sides on a square are examples. Defined numbers also are not subject to error.

Every measured number, however, no matter how carefully measured, has some degree of uncertainty. What is the width of your desk? Is it 98.5 centimeters, 98.52 centimeters, 98.520 centimeters or 98.5201 centimeters? You cannot state its exact measurement with absolute certainty.

Uncertainty in Measurement

The uncertainty (the margin of error) in a measurement depends on the precision of the measuring device and the skill of the person who uses it. Uncertainty and error in laboratory measurements have a different meaning from "human error." The mistakes your lab partner may make in sloppy lab procedures are entirely different from the margin of error inherent in every measurement. Uncertainties that relate to the precision of your measuring instruments cannot be avoided.

Uncertainty in a measurement can be illustrated by the two different metersticks in Figure A.1. The measurements are of the length of a table top. Assuming that the zero end of the meterstick has been carefully and accurately positioned at the left end of the table, how long is the table?

The upper scale in the figure is marked off in centimeter intervals. Using this scale you can say with certainty that the length is between 82 and 83 centimeters. You can say further that it is closer to 82 centimeters than to 83 centimeters. You can estimate it to be 82.2 centimeters.

The lower scale has more subdivisions and has a greater precision because it is marked off in millimeters. With this meterstick you can say that

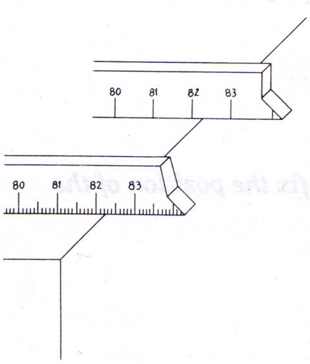
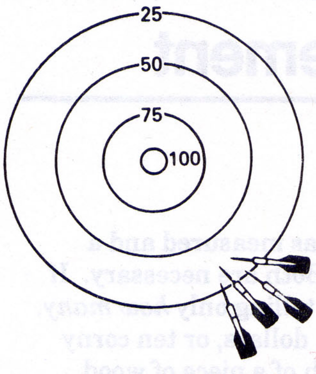


Figure A.1

the length is definitely between 82.2 and 82.3 centimeters, and you can estimate it to be 82.25 centimeters.

Note how both readings contain some digits that are exactly known, and one digit (the last one) that is estimated. Note also that the uncertainty in the reading of the lower meterstick is less than that of the top meterstick. The lower meterstick can give a reading to the hundredths place, and the top meterstick to the tenths place. The lower meterstick is more precise that the top one.

No measurements are exact. Measurements convey two kinds of information (1) the magnitude of the measurement and (2) the precision of the measurement. The location of the decimal point and the number value gives the magnitude. The precision is indicated by the number of significant figures recorded.

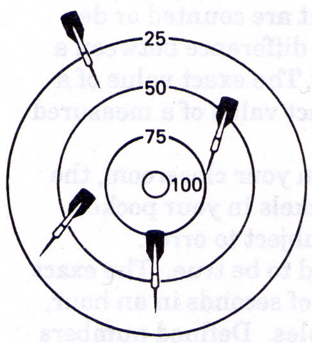


GOOD PRECISION BUT POOR ACCURACY

Significant Figures

Significant figures are the digits in any measurement that are known with certainty plus one digit that is estimated and hence is uncertain. The measurement 82.2 centimeters (made with the top meterstick in Figure A.1) has three significant figures, and the measurement 82.25 centimeters (made with the lower meterstick) has four significant figures. The right-most digit is always an estimated digit. Only one estimated digit is ever recorded as part of a measurement. It would be incorrect to report that in Figure A.1 the length of the table as measured with the lower meterstick is 82.253 centimeters. This five-significant-figure value would have two estimated digits (the 5 and 3) and would be incorrect because it indicates a precision greater that the meterstick can obtain.

Standard rules have been developed for writing and using signifcant figures, both in measurements and in values calculated from measurements.



POOR PRECISION AND POOR ACCURACY

Rule 1

In numbers that do not contain zeros, all the digits are significant.

EXAMPLES:

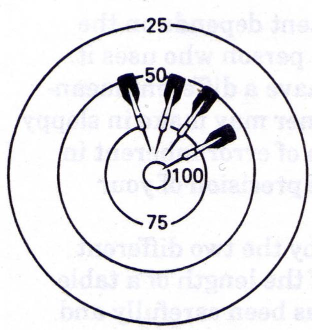
3.1428	five significant figures
3.14	three significant figures
469	three significant figures

Rule 2

All zeros between significant digits are significant.

EXAMPLES:

7.053	four significant figures
7053	four significant figures
302	three significant figures



GOOD PRECISION AND GOOD ACCURACY

Rule 3

Zeros to the left of the first nonzero digit serve only to fix the position of the decimal point and are not significant.

EXAMPLES:

0.0056	two significant figures
0.0789	three significant figures
0.000001	one significant figure

Rule 4

In a number with digits to the right of the decimal point, zeros to the right of the last non-zero digit are significant.

EXAMPLES:

43	two significant figures
43.0	three significant figures
43.00	four significant figures
0.00200	three significant figures
0.40050	five significant figures

Rule 5

In a number that has no decimal point and that ends in one or more zeros (such as 3600), the zeros that end the number may or may not be significant.

The number is ambiguous in terms of significant figures. Before the number of significant figures can be specified, further information is needed about how the number was obtained. If it is a measured number, the zeros are not significant. If the number is a defined or counted number, all the digits are significant.

Confusion is avoided when numbers are expressed in scientific notation. All digits are taken to be significant when expressed this way.

EXAMPLES:

3.6×10^{-5}	two significant figures
3.60×10^{-5}	three significant figures
3.600×10^{-5}	four significant figures
2×10^{-5}	one significant figure
2.0×10^{-5}	two significant figures
2.00×10^{-5}	three significant figures

Rounding Off

A calculator displays eight or more digits. How do you round off such a display of digits to, say, three significant figures? Three simple rules govern the process of deleting unwanted (nonsignificant) digits from a calculator number.

Rule 1

If the first digit to the right of the last significant figure is less than 5, that digit and all the digits that follow it are simply dropped.

EXAMPLE:

54.234 rounded off to three significant figures becomes 54.2.

Rule 2

If the first digit to be dropped is a digit greater than 5, or if it is a 5 followed by digits other than zero, the excess digits are dropped and the last retained digit is increased in value by one unit.

EXAMPLE:

54.35, 54.359, and 54.3598 rounded off to three significant figures all become 54.4.

Rule 3

If the first digit to be dropped is a 5 not followed by any other digit, or if it is a 5 followed only by zeros, an odd-even rule is applied.

That is, if the last retained digit is even, its value is not changed, and the 5 and any zeros that follow are dropped. But if the last digit is odd, its value is increased by one. The intention of this odd-even rule is to average the effects of rounding off.

EXAMPLES:

54.2500 to three significant figures becomes 54.2.
54.3500 to three significant figures becomes 54.4.

Significant Figures and Calculated Quantities

Suppose that you measure the mass of a small wooden block to be 2 grams on a balance, and you find that its volume is 3 cubic centimeters by poking it beneath the surface of water in a graduated cylinder. The density of the piece of wood is its mass divided by its volume. If you divide 2 by 3 on your calculator, the reading on the display is 0.6666666. It would be incorrect to report that the density of the block of wood is 0.6666666 gram per cubic centimeter. To do so would be claiming a degree of precision that is not warranted. Your answer should be rounded off to a sensible number of significant figures.

The number of significant figures allowable in a calculated result depends on the number of significant figures in the data used to obtain the result and on the type of mathematical operation(s) used to obtain the result. There are separate rules for multiplication and division, and for addition and subtraction.

Multiplication and Division

For multiplication and division, an answer should have the number of significant figures found in the number with the fewest significant figures. For the density example, the answer would be rounded off to one significant figure, 0.7 gram per cubic centimeter. If the mass were measured to be 2.0 grams, and if the volume were still taken to be 3 cubic centimeters, then the answer would still be rounded off to 0.7 gram per cubic centimeter. If the mass were measured to be 2.0 and the volume 3.0 or 3.00 cubic centimeters, the answer would be rounded off to two significant figures: 0.67 gram per cubic centimeter.

Study the following examples. Assume that the numbers being multiplied or divided are measured numbers.

EXAMPLE A:

$$8.536 \times 0.47 = 4.01192 \text{ (calculator answer)}$$

The input with the fewest significant figures is 0.47, which has two significant figures. Therefore, the calculator answer 4.01192 must be rounded off to 4.0.

EXAMPLE B:

3840 x 285.3 = 13.45916 (calculator answer)

The input with the fewest significant figures is 3840, which has three significant figures. Therefore, the calculator answer 13.45916 must be rounded off to 13.5.

EXAMPLE C:

360.0 ÷ 3.000 = 12 (calculator answer)

Both inputs contain four significant figures. Therefore, the correct answer must also contain four significant figures, and the calculator answer 12 must be written as 12.00. In this case the calculator gave too few significant figures.

Addition and Subtraction

For addition or subtraction, the answer should not have digits beyond the last digit position common to all the numbers being added and subtracted. Study the following examples:

EXAMPLE A:

15
18.8
34.6
67.4 (calculator answer)

The last digit position common to all numbers is the units place. Therefore, the calculator answer of 67.4 must be rounded off to the units place to become 67.

EXAMPLE B:

20.02
20.002
20.0002
60.0222 (calculator answer)

The last digit position common to all numbers is the hundredths place. The answer should be rounded off to 60.02.

EXAMPLE C:

345.56 - 245.5 = 100.06 (calculator answer)

The last digit position common to both numbers in this subtraction is the tenths place. Therefore, the answer should be rounded off to 100.1.

Percentage Error

If your aunt told you that she had made $100 in the stock market, you would be more impressed if this gain were on a $100 investment than if it were on a $10,000 investment. In the first case she would have doubled her investment and made a 100% gain. In the second case should would have made a 1% gain.

In laboratory measurements it is the percentage difference that is important, not the size of the difference. Measuring something to within 1 centimeter may be good or poor, depending on the length of the object you are measuring. Measuring the length of a 10-centimeter pencil to +/– 1 centimeter is quite a bit different from measuring the length of a 100-meter track to the same +/– 1 centimeter. The measurement of the pencil shows a relative uncertainty of 10%. The track measurement is uncertain by only 1 part in 10,000, or 0.01%.

The relative uncertainty or relative margin of error in measurements, when expressed as a percentage, is often called the percentage of error. It tells by what percentage a quantity differs from a known accepted value as determined by skilled observers using high precision equipment. It is a measure of the accuracy of the method of measurement as well as the skill of the person making the measurement. The percentage of error is found by dividing the difference between the measured value and the accepted value of a quantity by the accepted value, and then multiplying this quotient by 100%.

$$\text{percentage error} = \frac{|\text{measured value} - \text{accepted value}|}{\text{accepted value}} \times 100\%$$

For example, suppose that the measured value of the acceleration of gravity is found to be 9.44 m/s^2. The accepted value is 9.81 m/s^2. The difference between these two values is (9.81 m/s^2) – (9.44 m/s^2), or 0.37 m/s^2.

$$\%\ \text{error} = \frac{0.37\ m/s^2}{9.81\ m/s^2}$$

$$= 3.77\%$$