Derivation: Velocity of Approach Equals Velocity of Recession

Provided the collision is totally elastic, the velocity of approach equals the velocity of recession, regardless of the masses and initial velocities.

Initial Conditions:
- Mass 1 has mass $m_1$, velocity $v_{1i}$
- Mass 2 has mass $m_2$, velocity $v_{2i}$
- $m_1$ collides elastically with $m_2$
- After the collision, $m_1$ moves with velocity $v_{1f}$ and $m_2$ with velocity $v_{2f}$

Conservation of Momentum:

$$p_i = p_f$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \quad \text{Equation (1)}$$

Conservation of Energy:

$$KE_i = KE_f$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad \text{Equation (2)}$$

After re-arranging Equation (2) so that $m_1$ and $m_2$ are on opposite sides, factoring the masses, and finally expressing the difference in the velocities as the difference of two squares, we get:

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$m_2 (v_{2f}^2 - v_{2i}^2) = m_1 (v_{1i}^2 - v_{1f}^2)$$

$$m_2 (v_{2f} - v_{2i})(v_{2f} + v_{2i}) = m_1 (v_{1i} - v_{1f})(v_{1i} + v_{1f}) \quad \text{Equation (3)}$$
Now we re-arrange Equation (1) so that masses are on opposites sides:

\[ m_2(v_{2f} - v_{2i}) = m_1(v_{1i} - v_{1f}) \]  
Equation (4)

Now we divide Equation (3) by Equation (4):

\[
\frac{m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i})}{m_2(v_{2f} - v_{2i})} = \frac{m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f})}{m_1(v_{1i} - v_{1f})}
\]

\[ v_{2f} + v_{2i} = v_{1i} + v_{1f} \]  
Equation (5)

By rearranging terms in Equation (5), we obtain the final form of the result.

\[ v_{2f} - v_{1f} = -(v_{2i} - v_{1i}) \]

Velocity of approach equals velocity of recession.